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GENERALIZED M-CONTINUOUS AND M-IRRESOLUTE MAPPINGS IN FERMATEAN NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract: In this paper, we introduce and investigate Fermatean Neutrosophic generalized M-continuous mappings and Fermatean Neutrosophic generalized M-irresolute mappings within the framework of Fermatean Neutrosophic topological spaces. We systematically study the fundamental topological properties of these newly defined classes of mappings, analyze their behavior under set-theoretic operations, and establish their interrelations with other well-known classes in Fermatean Neutrosophic topology. Several illustrative examples are provided to clarify the concepts and demonstrate the applicability of the proposed framework. Additionally, we examine the structural aspects and role of these mappings in preserving Fermatean Neutrosophic generalized M-closed sets. The results significantly enrich the theoretical foundation of Fermatean Neutrosophic topology by extending classical ideas of open and closed mappings, and provide potential applications in decision sciences and information systems where uncertainty and vagueness are inherent.

Keywords and Phrases: Fermatean Neutrosophic generalized M-closed, Fermatean Neutrosophic generalized M-continuous and Fermatean Neutrosophic generalized M-irresolute mappings.

2020 Mathematics Subject Classification: 54A40, 54D30, 03E72.

1. Introduction

In real-world decision-making and information processing, uncertainty, ambiguity, and hesitation are common and cannot always be captured adequately by classical models. To address this limitation, Zadeh [25] introduced fuzzy sets in 1965, followed by Atanassov [1], who proposed intuitionistic fuzzy sets (IFS's) by incorporating both membership and non-membership degrees. However, in many practical situations, the condition $\mu + \nu \leq 1$ is too restrictive, which motivates the search for more flexible models.

To overcome this drawback, Yager [24] introduced Pythagorean fuzzy sets (PFS's), where the sum of the squares of membership and non-membership degrees is bounded by one. Later, Senapati and Yager [14] proposed Fermatean fuzzy sets (FFS's), in which the sum of the cubes of membership and non-membership degrees is constrained by one. This generalization provides greater freedom in representing uncertainty and has proved more effective than IFS's and PFS's in several decision-making problems.

Smarandache [15] introduced neutrosophic sets (NS's), where truth, indeterminacy, and falsity are treated as independent components. This independence makes neutrosophic structures especially suitable for inconsistent, incomplete, and indeterminate information. Combining the flexibility of Fermatean fuzzy sets with the expressive power of neutrosophic sets led to the development of Fermatean neutrosophic sets, which offer a stronger framework for modeling complex uncertain information.

Recent studies show the usefulness of Fermatean fuzzy theory in a variety of applications. For example, a unified parametric divergence operator was developed for the Fermatean fuzzy environment and applied to machine learning and intelligent decision-making [26]. Fermatean fuzzy Archimedean Heronian mean-based aggregation models were proposed for estimating sustainable urban transport solutions [9]. Fermatean fuzzy Dombi generalized Maclaurin symmetric mean operators were introduced for prioritizing bulk materials handling technologies [10]. In addition, Fermatean fuzzy copula aggregation operators and similarity measures were used in a COPRAS-based framework for renewable energy source selection [7], MCDM models and techniques in [3]. These studies demonstrate the practical relevance of Fermatean fuzzy models in handling complex and uncertain decision problems.

From a topological viewpoint, several generalized open-set concepts have been introduced to study continuity and related properties in fuzzy and neutrosophic settings. Ekici [5] introduced e-open sets in general topology, and Seenivasan et al. [13] extended this idea to fuzzy e-open sets and fuzzy e-continuity. Vadivel et al. [4] further developed these ideas in intuitionistic fuzzy topological spaces. In neutrosophic topology, Vadivel et al. [17] introduced δ-open sets, and later Vadivel and collaborators [6, 18] investigated M-open sets in neutrosophic nano and neutrosophic soft topological spaces. More recently, related concepts have also been studied in Fermatean neutrosophic settings [19, 20, 21], but the mapping theory in this framework remains relatively underdeveloped.

Problem Statement and Gap.

Although several generalized open-set notions and continuity notions have been studied in fuzzy, intuitionistic fuzzy, neutrosophic, and Fermatean neutrosophic topological spaces, there is still no systematic investigation of Fermatean neutrosophic generalized M-continuous maps and Fermatean neutrosophic generalized M-irresolute maps in Fermatean neutrosophic topological spaces. This gap exists because most earlier works focused on open-set structures and aggregation-based decision models, while the behavior of mappings preserving these structures has not yet been fully explored in the Fermatean neutrosophic context.

Research Question.

How can Fermatean neutrosophic generalized M-continuous and generalized M-irresolute mappings be defined, and what are their fundamental properties, examples, and interrelations in Fermatean neutrosophic topological spaces?

To answer this question, in this paper we introduce Fermatean neutrosophic generalized M-continuous maps and Fermatean neutrosophic generalized M-irresolute maps in the framework of Fermatean neutrosophic topological spaces. We investigate their basic properties, study their behavior under set-theoretic operations, and establish their relationships with other classes of sets and mappings in this setting. Several illustrative examples are provided to clarify the concepts and to demonstrate their applicability. The results obtained strengthen the theoretical foundation of Fermatean neutrosophic topology and open new directions for future research in uncertainty modeling and decision-making.

2. Preliminaries

Definition 2.1. [14] *Let X be a universe of discourse. A Fermatean fuzzy set ($\mathfrak{F}\mathcal{F}s$) F in X is an object having the form $F = \{< x, \alpha_F(x), \beta_F(x) >: x \in X\}$ where $\alpha_F(x) : X \to [0,1]$ and $\beta_F(x) : X \to [0,1]$, including the condition $0 \leq (\alpha_F(x))^3 + (\beta_F(x))^3 \leq 1$, for all $x \in X$. The numbers $\alpha_F(x)$ and $\beta_F(x)$ denote, respectively, the degree of membership and the degree of non-membership of the element x in the set F. For any $\mathfrak{F}\mathcal{F}s$ F and $x \in X$, $\pi_F(x) = \sqrt[3]{1 - [(\alpha_F(x))^3 - (\beta_F(x))^3]}$ is identified as the degree of indeterminacy of x to F. In the interest of simplicity, we shall mention the symbol $F = (\alpha_F, \beta_F)$ for the $\mathfrak{F}\mathcal{F}s$ $F = \{< x, \alpha_F(x), \beta_F(x) >: x \in X\}$.*

Definition 2.2. [12] *Let X be a non-empty set. A neutrosophic set (briefly, Ns) L is an object having the form $L = \{\langle x, \mu_L(x), \nu_L(x), \sigma_L(x)\rangle : x \in X\}$ where $\mu_L \to [0,1]$ denote the degree of membership function, $\nu_L \to [0,1]$ denote the degree of indeterminacy function and $\sigma_L \to [0,1]$ denote the degree of non-membership function respectively of each element $x \in X$ to the set L and $0 \leq \mu_L(x) + \nu_L(x) + \sigma_L(x) \leq 3$ for each $x \in X$.*

Definition 2.3. [11] *A neutrosophic topology (briefly, Nt) on a non-empty set X is a family τ_N of neutrosophic subsets of X satisfying*

(i) 0_N, $1_N \in \tau_N$.

(ii) $L_1 \cap L_2 \in \tau_N$ *for any* $L_1, L_2 \in \tau_N$.

(iii) $\bigcup L_a \in \tau_N, \forall \ L_a : a \in A \subseteq \tau_N$.

Then (X, τ_N) is called a neutrosophic topological space (briefly, Nts) in X. The τ_N elements are called neutrosophic open sets (briefly, Nos) in X. A Ns C is called a neutrosophic closed sets (briefly, Ncs) iff its complement C^c is Nos.

Definition 2.4. [16] *Let X be a non-empty set. A Fermatean neutrosophic set (briefly, $\mathfrak{F}\mathfrak{N}s$) L is an object having the form $L = \{\langle x, \mu_L(x), \nu_L(x), \sigma_L(x)\rangle : x \in X\}$ where $\mu_L \to [0,1]$ denote the degree of membership function, $\nu_L \to [0,1]$ denote the degree of indeterminacy function and $\sigma_L \to [0,1]$ denote the degree of non-membership function respectively of each element $x \in X$ to the set L such that $0 \leq (\mu_L(x))^3 + (\sigma_L(x))^3 \leq 1$ and $0 \leq (\nu_L(x))^3 \leq 1$. Then $0 \leq (\mu_L(x))^3 + (\nu_L(x))^3 +$*

$(\sigma_L(x))^3 \leq 2$ *for all* $x \in X$. *Here* $\mu_L(x)$ *and* $\sigma_L(x)$ *are dependent components and* $\nu_L(x)$ *is an independent component.*

The definitions of $1_{\mathfrak{FN}}$ and $0_{\mathfrak{FN}}$ that will be needed before proceeding to set operations will be given. In [11], possible definitions of $1_{\mathfrak{FN}}$ and $0_{\mathfrak{FN}}$ neutrosophic sets are given. In this paper, the theory will be constructed by defining $0_{\mathfrak{FN}}$ and $1_{\mathfrak{FN}}$ Fermatean neutrosophic sets in a single way. $0_{\mathfrak{FN}}$ and $1_{\mathfrak{FN}}$ are defined as $0_{\mathfrak{FN}} = \{(x, 0, 0, 1) : x \in X\}$ and $1_{\mathfrak{FN}} = \{(x, 1, 1, 0) : x \in X\}$ Now, the union, intersection and complement definitions necessary for the definition of the topological space will be given. These definitions are given in several different ways in classical neutrosophic spaces in [2]; to avoid confusion here, only one method will be given for sets with Fermatean structure, and this method is different from the method chosen in [11].

Definition 2.5. [16] *Let* X *be a non-empty set & the* $\mathfrak{FNs}$*'s* L *&* M *in the form* $L = \{\langle x, \mu_L(x), \nu_L(x), \sigma_L(x)\rangle : x \in X\}$, $M = \{\langle x, \mu_M(x), \nu_M(x), \sigma_M(x)\rangle : x \in X\}$, *then*

(i) $0_{\mathfrak{FN}} = \langle x, 0, 0, 1\rangle$ *and* $1_{\mathfrak{FN}} = \langle x, 1, 1, 0\rangle$,

(ii) $L \subseteq M$ *iff* $\mu_L(x) \leq \mu_M(x)$, $\nu_L(x) \leq \nu_M(x)$ & $\sigma_L(x) \geq \sigma_M(x) : x \in X$,

(iii) $L = M$ *iff* $L \subseteq M$ *and* $M \subseteq L$,

(iv) $1_{\mathfrak{FN}} - L = \{\langle x, \sigma_L(x), 1_{\mathfrak{FN}} - \nu_L(x), \mu_L(x)\rangle : x \in X\} = L^c$ *or* $C(L)$,

(v) $L \cup M = \{\langle x, \max(\mu_L(x), \mu_M(x)), \max(\nu_L(x), \nu_M(x)), \min(\sigma_L(x), \sigma_M(x))\rangle : x \in X\}$,

(vi) $L \cap M = \{\langle x, \min(\mu_L(x), \mu_M(x)), \min(\nu_L(x), \nu_M(x)), \max(\sigma_L(x), \sigma_M(x))\rangle : x \in X\}$.

Definition 2.6. [16] *Let*

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}, \quad B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle : x \in X\}$$

be two Fermatean Neutrosophic Sets ($\mathfrak{FN}$s's) on a universe X, *where for all* $x \in X$, $T_A(x), I_A(x), F_A(x), T_B(x), I_B(x), F_B(x) \in [0, 1]$ *and* $T_A^3(x) + I_A^3(x) + F_A^3(x) \leq 1$, $T_B^3(x) + I_B^3(x) + F_B^3(x) \leq 1$. *Then the difference of* A *and* B, *denoted by* $A - B$, *is defined as a new* $\mathfrak{FN}$s C, *where for each* $x \in X$,

$$C(x) = A(x) - B(x) = \langle x,\ T_C(x),\ I_C(x),\ F_C(x)\rangle$$

with:

$$T_C(x) = \max(0,\ T_A(x) - T_B(x)),$$
$$I_C(x) = \max(1,\ I_A(x) + I_B(x)),$$
$$F_C(x) = \min(1,\ F_A(x) + T_B(x)).$$

To ensure that the Fermatean condition is preserved, if

$$T_C^3(x) + I_C^3(x) + F_C^3(x) > 1,$$

we normalize the components as follows:

$$T'_C(x) = \frac{T_C(x)}{M}, \quad I'_C(x) = \frac{I_C(x)}{M}, \quad F'_C(x) = \frac{F_C(x)}{M},$$

where

$$M = \left(T_C^3(x) + I_C^3(x) + F_C^3(x)\right)^{1/3}.$$

Definition 2.7. [8] *A Fermatean neutrosophic topology (briefly, $\mathfrak{FN}t$) on a non-empty set X is a family $\tau_{\mathfrak{FN}}$ of Fermatean neutrosophic subsets of X satisfying*

(i) $0_{\mathfrak{FN}}, 1_{\mathfrak{FN}} \in \tau_{\mathfrak{FN}}$,

(ii) $L_1 \cap L_2 \in \tau_{\mathfrak{FN}}$ *for any* $L_1, L_2 \in \tau_{\mathfrak{FN}}$,

(iii) $\bigcup L_a \in \tau_{\mathfrak{FN}}, \forall\ L_a : a \in A \subseteq \tau_{\mathfrak{FN}}$.

Then $(X, \tau_{\mathfrak{FN}})$ is called a Fermatean neutrosophic topological space (briefly, $\mathfrak{FN}ts$) in X. The $\tau_{\mathfrak{FN}}$ elements are called Fermatean neutrosophic open sets (briefly, $\mathfrak{FN}os$) in X. A $\mathfrak{FN}s$ C is called a Fermatean neutrosophic closed sets (briefly, $\mathfrak{FN}cs$) iff its complement C^c is $\mathfrak{FN}os$.

Definition 2.8. [8] *Let $(X, \tau_{\mathfrak{FN}})$ be $\mathfrak{FN}ts$ on X and L be an $\mathfrak{FN}s$ on X, then the Fermatean neutrosophic interior of L (briefly, $\mathfrak{FN}int(L)$) and the Fermatean neutrosophic closure of L (briefly, $\mathfrak{FN}cl(L)$) are defined as*

$$\mathfrak{FN}int(L) = \bigcup\{I : I \subseteq L \ \&\ I \text{ is a } \mathfrak{FN}os \text{ in } X\}$$

$$\mathfrak{FN}cl(L) = \bigcap\{I : L \subseteq I \ \&\ I \text{ is a } \mathfrak{FN}cs \text{ in } X\}.$$

Theorem 2.1. [8] *Let L be an $\mathfrak{FN}s$ on X. In this case, the following four properties hold:*

(i) $\mathfrak{FN}cl(L)$ *is a closed Fermatean neutrosophic set,*

(ii) $\mathfrak{FN}cl(1_{\mathfrak{FN}}) = 1_{\mathfrak{FN}}$, $\mathfrak{FN}cl(0_{\mathfrak{FN}}) = 0_{\mathfrak{FN}}$,

(iii) $\mathfrak{FN}int(L)$ *is an open Fermatean neutrosophic set.*

(iv) $\mathfrak{FN}int(1_{\mathfrak{FN}}) = 1_{\mathfrak{FN}}$, $\mathfrak{FN}int(0_{\mathfrak{FN}}) = 0_{\mathfrak{FN}}$.

Lemma 2.1. [8] *For any Fermatean neutrosophic set* A *in* $(X, \tau_{\mathfrak{FN}})$*, we have* $C(\mathfrak{FN}int\ (A)) = \mathfrak{FN}cl(C(A))$ *and* $C(\mathfrak{FN}cl(A)) = \mathfrak{FN}int(C(A))$*. Here* $C(A)$ *or* $\overline{A}$ *denotes complement of* A*.*

Definition 2.9. [23] *Let* $(X, \tau_{\mathfrak{FN}})$ *be an* $\mathfrak{FN}ts$ *and* A *be an* $\mathfrak{FN}s$*. Then* A *is said to be an Fermatean neutrosophic (i) regular open set (*$\mathfrak{FN}ros$ *in short) if* $A = \mathfrak{FN}int(\mathfrak{FN}cl(A))$*. (ii) regular closed set (*$\mathfrak{FN}rcs$ *in short) if* $A = \mathfrak{FN}cl(\mathfrak{FN}int(A))$*. By Lemma 2.1, it follows that* A *is an* $\mathfrak{FN}ros$ *iff* $\bar{A}$ *is an* $\mathfrak{FN}rcs$*.*

Definition 2.10. [23] *Let* $(X, \tau_{\mathfrak{FN}})$ *be an* $\mathfrak{FN}ts$ *and* $A = \{< a, \mu_A(a), \nu_A(a), \sigma_A(a) > |a \in X\}$ *be an* $\mathfrak{FN}s$ *in* X*. Then the* δ*-interior and the* δ*-closure of* A *are denoted by* $\mathfrak{FN}\delta int(A)$ *and* $\mathfrak{FN}\delta cl(A)$ *and are defined as follows.* $\mathfrak{FN}\delta int(A) = \cup\{G|G$ *is an* $\mathfrak{FN}ros$ *and* $G \subseteq A\}$, $\mathfrak{FN}\delta cl(A) = \cap\{K|K$ *is an* $\mathfrak{FN}rcs$ *and* $A \subseteq K\}$.

Definition 2.11. [23] *Let* $(X, \tau_{\mathfrak{FN}})$ *be an* $\mathfrak{FN}ts$ *and* $A = \{< a, \mu_A(a), \nu_A(a), \sigma_A(a) > |a \in X\}$ *be an* $\mathfrak{FN}s$ *in* X*. A set* A *is said to be* $\mathfrak{FN}$

(i) δ*-open set (briefly,* $\mathfrak{FN}\delta os$*) if* $A = \mathfrak{FN}\delta int(A)$*,*

(ii) δ*-pre open set (briefly,* $\mathfrak{FN}\delta\mathcal{P}os$*) if* $A \subseteq \mathfrak{FN}int(\mathfrak{FN}\delta cl(A))$*.*

(iii) δ*-semi open set (briefly,* $\mathfrak{FN}\delta\mathcal{S}os$*) if* $A \subseteq \mathfrak{FN}cl(\mathfrak{FN}\delta int(A))$*.*

(iv) δ *(resp.* δ*-pre and* δ*-semi) dense if* $\mathfrak{FN}\delta cl(A)$ *(resp.* $\mathfrak{FN}\delta\mathcal{P}cl(A)$*, and* $\mathfrak{FN}\delta\mathcal{S}cl\ (A)) = 1_{\mathfrak{FN}}$*.*

The complement of an $\mathfrak{FN}\delta os$ (resp. $\mathfrak{FN}\delta\mathcal{P}os$ and $\mathfrak{FN}\delta\mathcal{S}os$) is called an $\mathfrak{FN}\delta$ (resp. $\mathfrak{FN}\delta\mathcal{P}$ and $\mathfrak{FN}\delta\mathcal{S}$) closed set (briefly, $\mathfrak{FN}\delta cs$ (resp. $\mathfrak{FN}\delta\mathcal{P}cs$ and $\mathfrak{FN}\delta\mathcal{S}cs$)) in X.

The family of all $\mathfrak{FN}\delta os$ (resp. $\mathfrak{FN}\delta cs$, $\mathfrak{FN}\delta\mathcal{P}os$, $\mathfrak{FN}\delta\mathcal{P}cs$, $\mathfrak{FN}\delta\mathcal{S}os$ and $\mathfrak{FN}\delta\mathcal{S}cs$) of X is denoted by $\mathfrak{FN}\delta OS(X)$, (resp. $\mathfrak{FN}\delta CS(X)$, $\mathfrak{FN}\delta\mathcal{P}OS(X)$, $\mathfrak{FN}\delta\mathcal{P}CS(X)$, $\mathfrak{FN}\delta\mathcal{S}OS(X)$ and $\mathfrak{FN}\delta\mathcal{S}CS(X)$).

Definition 2.12. [23] *Let* $(X, \tau_{\mathfrak{FN}})$ *be an* $\mathfrak{FN}ts$ *and* $A = \{< a, \mu_A(a), \nu_A(a), \sigma_A(a) > |a \in X\}$ *be an* $\mathfrak{FN}s$ *in* X*. Then the* $\mathfrak{FN}\delta$ *(resp.* $\mathfrak{FN}\delta$*-pre and* $\mathfrak{FN}\delta$*-semi)-interior and the* $\mathfrak{FN}\delta$ *(resp.* $\mathfrak{FN}\delta$*-pre and* $\mathfrak{FN}\delta$*-semi)-closure of* A *are denoted by* $\mathfrak{FN}\delta int(A)$ *(resp.* $\mathfrak{FN}\delta\mathcal{P}int(A)$*,* $\mathfrak{FN}\delta\mathcal{S}int(A)$ *and the* $\mathfrak{FN}\delta cl(A)$ *(resp.* $\mathfrak{FN}\delta\mathcal{P}cl(A)$ *and* $\mathfrak{FN}\delta\mathcal{S}cl\ (A))$ *and are defined as follows:*

$\mathfrak{FN}\delta int(A)$ *(resp.* $\mathfrak{FN}\delta\mathcal{P}int(A)$ *and* $\mathfrak{FN}\delta\mathcal{S}int(A)) = \cup\{G|G$ *in a* $\mathfrak{FN}\delta os$ *(resp.* $\mathfrak{FN}\delta\mathcal{P}os$ *and* $\mathfrak{FN}\delta\mathcal{S}os$*) and* $G \subseteq A\}$ *and* $\mathfrak{FN}\delta cl(A)$ *(resp.* $\mathfrak{FN}\delta\mathcal{P}cl(A)$ *and* $\mathfrak{FN}\delta\mathcal{S}cl(A)) = \cap\{K|K$ *is an* $\mathfrak{FN}\delta cs$ *(resp.* $\mathfrak{FN}\delta\mathcal{P}cs$ *and* $\mathfrak{FN}\delta\mathcal{S}cs$*) and* $A \subseteq K\}$.

3. Fermatean neutrosophic generalized M - continuous

In this section, we study the concepts of Fermatean neutrosophic generalized (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M)-continuous and some of their basic properties.

Definition 3.1. *Let* $(X, \tau_{\mathfrak{FN}})$ *be a* $\mathfrak{FN}ts$ *and* S *be a* $\mathfrak{FN}s$ *in* X. *A set* S *is said to be* $\mathfrak{FN}$

(i) θ*-interior of* S *(briefly,* $\mathfrak{FN}\theta int(S)$*) is defined by*

$$\mathfrak{FN}\theta int(S) = \bigcup\{\mathfrak{FN}int(T) : T \subseteq S \ \& \ T \text{ is a } \mathfrak{FN}cs \text{ in } X\}.$$

(ii) θ*-open set (briefly,* $\mathfrak{FN}\theta os$*) if* $S = \mathfrak{FN}\theta int(S)$.

(iii) θ *-semi open set (briefly,* $\mathfrak{FN}\theta\mathcal{S}os$*) if* $S \subseteq \mathfrak{FN}cl(\mathfrak{FN}\theta int(S))$.

(iv) M*-open set (briefly,* $\mathfrak{FN}Mos$*) if* $S \subseteq \mathfrak{FN}cl(\mathfrak{FN}\theta int(S)) \cup \mathfrak{FN}int(\mathfrak{FN}\delta cl(S))$.

The complement of a $\mathfrak{FN}Mos$ (resp. $\mathfrak{FN}\theta os$ & $\mathfrak{FN}\theta\mathcal{S}os$) is called an $\mathfrak{FN}M$ (resp. $\mathfrak{FN}\theta$ & $\mathfrak{FN}\theta\mathcal{S}$) closed set (briefly, $\mathfrak{FN}Mcs$ (resp. $\mathfrak{FN}\theta cs$ & $\mathfrak{FN}\theta\mathcal{S}cs$)) in X.

The family of all $\mathfrak{FN}\theta os$ (resp. $\mathfrak{FN}\theta cs$, $\mathfrak{FN}\theta\mathcal{S}os$, $\mathfrak{FN}\theta\mathcal{S}cs$, $\mathfrak{FN}Mos$ and $\mathfrak{FN}Mcs$) of X is denoted by $\mathfrak{FN}\theta OS(X)$ (resp. $\mathfrak{FN}\theta CS(X)$, $\mathfrak{FN}\theta\mathcal{S}OS(X)$, $\mathfrak{FN}\theta\mathcal{S}CS(X)$, $\mathfrak{FN}MOS(X)$ and $\mathfrak{FN}MCS(X)$).

Definition 3.2. *Let* $(X, \tau_{\mathfrak{FN}})$ *be a* $\mathfrak{FN}ts$ *and* S *be a* $\mathfrak{FN}s$ *in* X. *Then the* $\mathfrak{FN}$

(i) M*-interior (resp.* $\mathfrak{FN}\theta$*-interior and* $\mathfrak{FN}\theta$*-semi interior) of* S *(briefly,* $\mathfrak{FN}Mint(S)$ *(resp.* $\mathfrak{FN}\theta int(S)$, $\mathfrak{FN}\theta\ \mathcal{S}int(S)$*) is defined by* $\mathfrak{FN}Mint(S)$ *(resp.* $\mathfrak{FN}\theta int(S)$ *and* $\mathfrak{FN}\theta\mathcal{S}int(S)) = \cup\{T : T \subseteq S$ *and* T *is a* $\mathfrak{FN}Mos$ *(resp.* $\mathfrak{FN}\theta os$ *and* $\mathfrak{FN}\theta\mathcal{S}os$*) in* $X\}$.

(ii) M*-closure (resp.* θ*-closure and* θ*-semi closure) of* S *(briefly,* $\mathfrak{FN}Mcl(S)$ *(resp.* $\mathfrak{FN}\theta cl(S)$ & $\mathfrak{FN}\theta\mathcal{S}cl(S)$*) is defined by* $\mathfrak{FN}Mcl(S)$ *(resp.* $\mathfrak{FN}\theta cl(S)$ *and* $\mathfrak{FN}\theta\mathcal{S}cl(S)) = \cap\{T : S \subseteq T$ *and* T *is a* $\mathfrak{FN}Mcs$ *(resp.* $\mathfrak{FN}\theta cs$ *and* $\mathfrak{FN}\theta\mathcal{S}cs$*) in* $X\}$.

Definition 3.3. *Let* $(X, \tau_{\mathfrak{FN}})$ *be an* $\mathfrak{FN}ts$ *and* K *be an* $\mathfrak{FN}s$. *Then* K *is said to be an Fermatean neutrosophic generalized*

(i) closed, (briefly, $\mathfrak{FN}gc$*) set if* $\mathfrak{FN}cl(K) \subseteq V$ *whenever* $K \subseteq V$ *and* V *is* $\mathfrak{FN}o$ *in* X,

(ii) *θ-closed, (briefly, $\mathfrak{FN}g\theta c$) set if $\mathfrak{FN}\theta cl(K) \subseteq V$ whenever $K \subseteq V$ and V is $\mathfrak{FN}o$ in X,*

(iii) *$\theta\mathcal{S}$-closed, (briefly, $\mathfrak{FN}g\theta\mathcal{S}c$) set if $\mathfrak{FN}\theta\mathcal{S}cl(K) \subseteq V$ whenever $K \subseteq V$ and V is $\mathfrak{FN}o$ in X,*

(iv) *$\delta\mathcal{P}$-closed, (briefly, $\mathfrak{FN}g\delta\mathcal{P}c$) set if $\mathfrak{FN}\delta\mathcal{P}cl(K) \subseteq V$ whenever $K \subseteq V$ and V is $\mathfrak{FN}o$ in X,*

(v) *M-closed, (briefly, $\mathfrak{FN}gMc$) set if $\mathfrak{FN}Mcl(K) \subseteq V$ whenever $K \subseteq V$ and V is $\mathfrak{FN}o$ in X.*

The collection of all $\mathfrak{FN}gc$ (resp. $\mathfrak{FN}g\theta c$, $\mathfrak{FN}g\theta\mathcal{S}c$, $\mathfrak{FN}g\delta\mathcal{P}c$ and $\mathfrak{FN}gMc$) sets of X is denoted by $\mathfrak{FN}GC(X)$ (resp. $\mathfrak{FN}G\theta C(X)$, $\mathfrak{FN}G\theta\mathcal{S}C(X)$, $\mathfrak{FN}G\delta\mathcal{P}C(X)$ and $\mathfrak{FN}GMC(X)$).

Theorem 3.1. *Let $(X, \tau_{\mathfrak{FN}})$ be a $\mathfrak{FN}ts$. Then the following statements are hold but the converse does not true.*

(i) *Every $\mathfrak{FN}g\theta c$ set is $\mathfrak{FN}gc$ set,*

(ii) *Every $\mathfrak{FN}g\theta c$ set is $\mathfrak{FN}g\delta\mathcal{P}c$ set,*

(iii) *Every $\mathfrak{FN}g\theta c$ set is $\mathfrak{FN}g\theta\mathcal{S}c$ set,*

(iv) *Every $\mathfrak{FN}g\delta\mathcal{P}c$ set is $\mathfrak{FN}gMc$ set,*

(v) *Every $\mathfrak{FN}g\theta\mathcal{S}c$ set is $\mathfrak{FN}gMc$ set.*

Proof. (v) Assume that K is a $\mathfrak{FN}g\theta\mathcal{S}c$ in $(X, \tau_{\mathfrak{FN}})$ and Let G be a $\mathfrak{FN}o$ in $(X, \tau_{\mathfrak{FN}})$ $K \subseteq G$, $\mathfrak{FN}g\theta\mathcal{S}cl(K) \subseteq G$. Since, every $\mathfrak{FN}\theta\mathcal{S}c$ set is $\mathfrak{FN}Mc$. That is $\mathfrak{FN}Mcl(K) \subseteq \mathfrak{FN}g\theta\mathcal{S}cl(K) \subseteq G$. Therefore, K is $\mathfrak{FN}gMc$.

The rest of the results can be proved in same manner.

Definition 3.4. *Let $(X, \tau_{\mathfrak{FN}})$ be an $\mathfrak{FN}ts$ and K be an $\mathfrak{FN}s$. Then the Fermatean neutrosophic generalized (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M)-closure of K is the intersection of all $\mathfrak{FN}gc$ (resp. $\mathfrak{FN}g\theta c$, $\mathfrak{FN}g\theta\mathcal{S}c$, $\mathfrak{FN}g\delta\mathcal{P}c$ and $\mathfrak{FN}gMc$) sets containing K and denoted by $\mathfrak{FN}gcl(K)$ (resp. $\mathfrak{FN}g\theta cl(K)$, $\mathfrak{FN}g\theta\mathcal{S}cl(K)$, $\mathfrak{FN}g\delta\mathcal{P}cl(K)$ and $\mathfrak{FN}gMcl(K)$).*

Definition 3.5. *A map $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ is called Fermatean neutrosophic generalized*

(i) *continuous (briefly, $\mathfrak{FN}gCts$) if the inverse image of every $\mathfrak{FN}cs$ in $(Y, \sigma_{\mathfrak{FN}})$ is a $\mathfrak{FN}gcs$ in $(X, \tau_{\mathfrak{FN}})$,*

(ii) *θ-continuous (briefly, $\mathfrak{FN}g\theta Cts$) if the inverse image of every $\mathfrak{FN}cs$ in $(Y, \sigma_{\mathfrak{FN}})$ is a $\mathfrak{FN}g\theta cs$ in $(X, \tau_{\mathfrak{FN}})$,*

(iii) *$\theta\mathcal{S}$-continuous (briefly, $\mathfrak{FN}g\theta\mathcal{S}Cts$) if the inverse image of every $\mathfrak{FN}cs$ in $(Y, \sigma_{\mathfrak{FN}})$ is a $\mathfrak{FN}g\theta\mathcal{S}cs$ in $(X, \tau_{\mathfrak{FN}})$,*

(iv) *$\delta\mathcal{P}$-continuous (briefly, $\mathfrak{FN}g\delta\mathcal{P}Cts$) if the inverse image of every $\mathfrak{FN}cs$ in $(Y, \sigma_{\mathfrak{FN}})$ is a $\mathfrak{FN}g\delta\mathcal{P}cs$ in $(X, \tau_{\mathfrak{FN}})$,*

(v) *M-continuous (briefly, $\mathfrak{FN}gMCts$) if the inverse image of every $\mathfrak{FN}cs$ in $(Y, \sigma_{\mathfrak{FN}})$ is a $\mathfrak{FN}gMcs$ in $(X, \tau_{\mathfrak{FN}})$.*

Theorem 3.2. *Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ be a mapping. Then*

(i) *Every $\mathfrak{FN}g\theta Cts$ mapping is $\mathfrak{FN}gCts$,*

(ii) *Every $\mathfrak{FN}g\theta Cts$ mapping is $\mathfrak{FN}g\delta\mathcal{P}Cts$,*

(iii) *Every $\mathfrak{FN}g\theta Cts$ mapping is $\mathfrak{FN}g\theta\mathcal{S}Cts$,*

(iv) *Every $\mathfrak{FN}g\delta\mathcal{P}Cts$ mapping is $\mathfrak{FN}gMCts$,*

(v) *Every $\mathfrak{FN}g\theta\mathcal{S}Cts$ mapping is $\mathfrak{FN}gMCts$.*

But the converse does not true.

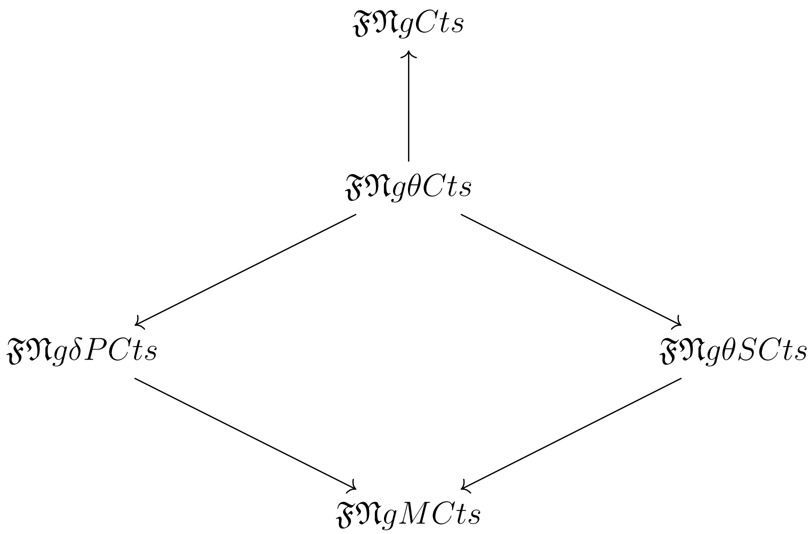
Proof. We prove only (iv) and (v), the others are similar.

(iv) Let λ be a $\mathfrak{FN}cs$ in Y. Since f is $\mathfrak{FN}g\delta\mathcal{P}Cts$, $f^{-1}(\lambda)$ is a $\mathfrak{FN}g\delta\mathcal{P}cs$ in X. Since every $\mathfrak{FN}g\delta\mathcal{P}cs$ is a $\mathfrak{FN}gMcs$, $f^{-1}(\lambda)$ is a $\mathfrak{FN}gMcs$ in X. Hence f is a $\mathfrak{FN}gMCts$.

(v) Let λ be a $\mathfrak{FN}cs$ in Y. Since f is $\mathfrak{FN}g\theta\mathcal{S}Cts$, $f^{-1}(\lambda)$ is a $\mathfrak{FN}g\theta\mathcal{S}cs$ in X. Since every $\mathfrak{FN}g\theta\mathcal{S}cs$ is a $\mathfrak{FN}gMcs$, $f^{-1}(\lambda)$ is a $\mathfrak{FN}gMcs$ in X. Hence f is a $\mathfrak{FN}gMCts$.

Remark 3.1. *The implication diagram is obtained and none of the implications in the diagram are reversible as seen from the example.*

Example 3.1. Let $X = Y = \{a, b\}$ and the $\mathfrak{FN}s$'s A_1, A_2 and A_3 are defined as
$\mu_{A_1}(a) = 0.8,\ \nu_{A_1}(a) = 0.8,\ \sigma_{A_1}(a) = 0.1,$
$\mu_{A_1}(b) = 0.9,\ \nu_{A_1}(b) = 0.8,\ \sigma_{A_1}(b) = 0.2;$



Note: $A \rightarrow B$ denotes A implies B, but not conversely.

$\mu_{A_2}(a) = 0.6,\ \nu_{A_2}(a) = 0.7,\ \sigma_{A_2}(a) = 0.2,$
$\mu_{A_2}(b) = 0.5,\ \nu_{A_2}(b) = 0.7,\ \sigma_{A_2}(b) = 0.6;$
$\mu_{A_3}(a) = 0.1,\ \nu_{A_3}(a) = 0.2,\ \sigma_{A_3}(a) = 0.7,$
$\mu_{A_3}(b) = 0.2,\ \nu_{A_3}(b) = 0.3,\ \sigma_{A_3}(b) = 0.8.$

Let $\tau_{\mathfrak{FN}} = \sigma_{\mathfrak{FN}} = \{0_{\mathfrak{FN}},\ 1_{\mathfrak{FN}},\ A_1,\ A_2,\ A_3\}$ be a $\mathfrak{FN}ts$ on X. Let $f : (X, \tau_{\mathfrak{FN}}) \rightarrow (Y, \sigma_{\mathfrak{FN}})$ be an identity mapping, then f is (i) $\mathfrak{FN}gCts$ but not $\mathfrak{FN}g\theta Cts$, the set A_1^c is a $\mathfrak{FN}cs$ in Y but $f^{-1}(A_1^c)$ is not $\mathfrak{FN}g\theta cs$ in X, (ii) $\mathfrak{FN}gMCts$ but not $\mathfrak{FN}g\theta\mathcal{S}Cts$, the set A_2^c is a $\mathfrak{FN}cs$ in Y but $f^{-1}(A_2^c)$ is not $\mathfrak{FN}g\theta\mathcal{S}cs$ in X, (iii) $\mathfrak{FN}g\delta\mathcal{P}Cts$ but not $\mathfrak{FN}g\theta Cts$, the set A_1^c is a $\mathfrak{FN}cs$ in Y but $f^{-1}(A_1^c)$ is not $\mathfrak{FN}g\theta cs$ in X.

Example 3.2. Let $X = Y = \{a, b\}$ and the $\mathfrak{FN}s$'s A_1, A_2 and A_3 are defined as
$\mu_{A_1}(a) = 0.8,\ \nu_{A_1}(a) = 0.8,\ \sigma_{A_1}(a) = 0.1,$
$\mu_{A_1}(b) = 0.9,\ \nu_{A_1}(b) = 0.8,\ \sigma_{A_1}(b) = 0.2;$
$\mu_{A_2}(a) = 0.6,\ \nu_{A_2}(a) = 0.7,\ \sigma_{A_2}(a) = 0.2,$
$\mu_{A_2}(b) = 0.5,\ \nu_{A_2}(b) = 0.7,\ \sigma_{A_2}(b) = 0.6;$
$\mu_{A_3}(a) = 0.1,\ \nu_{A_3}(a) = 0.2,\ \sigma_{A_3}(a) = 0.7,$
$\mu_{A_3}(b) = 0.2,\ \nu_{A_3}(b) = 0.3,\ \sigma_{A_3}(b) = 0.8.$

Let $\tau_{\mathfrak{FN}} = \{0_{\mathfrak{FN}},\ 1_{\mathfrak{FN}},\ A_1,\ A_2,\ A_3\}$, $\sigma_{\mathfrak{FN}} = \{0_{\mathfrak{FN}},\ 1_{\mathfrak{FN}},\ A_1\}$ be a $\mathfrak{FN}ts$ on X. Let $f : (X, \tau_{\mathfrak{FN}}) \rightarrow (Y, \sigma_{\mathfrak{FN}})$ be an identity mapping, then f is $\mathfrak{FN}g\theta SCts$ but not $\mathfrak{FN}g\theta Cts$, the set A_1^c is a $\mathfrak{FN}cs$ in Y but $f^{-1}(A_1^c)$ is not $\mathfrak{FN}g\theta cs$ in X.

Example 3.3. Let $X = Y = \{a, b\}$ and the $\mathfrak{FN}s$'s A_1, A_2, A_3 and A_4 are defined as
$\mu_{A_1}(a) = 0.8,\ \nu_{A_1}(a) = 0.8,\ \sigma_{A_1}(a) = 0.1,$
$\mu_{A_1}(b) = 0.9,\ \nu_{A_1}(b) = 0.8,\ \sigma_{A_1}(b) = 0.2;$
$\mu_{A_2}(a) = 0.6,\ \nu_{A_2}(a) = 0.7,\ \sigma_{A_2}(a) = 0.2,$
$\mu_{A_2}(b) = 0.5,\ \nu_{A_2}(b) = 0.7,\ \sigma_{A_2}(b) = 0.6;$
$\mu_{A_3}(a) = 0.1,\ \nu_{A_3}(a) = 0.2,\ \sigma_{A_3}(a) = 0.7,$
$\mu_{A_3}(b) = 0.2,\ \nu_{A_3}(b) = 0.3,\ \sigma_{A_3}(b) = 0.8;$
$\mu_{A_4}(a) = 0.2,\ \nu_{A_4}(a) = 0.3,\ \sigma_{A_4}(a) = 0.6,$
$\mu_{A_4}(b) = 0.6,\ \nu_{A_4}(b) = 0.3,\ \sigma_{A_4}(b) = 0.5.$

Let $\tau_{\mathfrak{FN}} = \{0_{\mathfrak{FN}},\ 1_{\mathfrak{FN}},\ A_1,\ A_2,\ A_3\}$, $\sigma_{\mathfrak{FN}} = \sigma_{\mathfrak{FN}} = \{0_{\mathfrak{FN}},\ 1_{\mathfrak{FN}},\ A_4\}$ be a $\mathfrak{FN}ts$ on X. Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ be an identity mapping, then f is $\mathfrak{FN}g\delta\mathcal{P}Cts$ but not $\mathfrak{FN}gMCts$, the set A_4^c is a $\mathfrak{FN}cs$ in Y but $f^{-1}(A_4^c)$ is not $\mathfrak{FN}g\delta\mathcal{P}cs$ in X.

Theorem 3.3. *A map $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ is $\mathfrak{FN}gMCts$ iff the inverse image of each $\mathfrak{FN}os$ in Y is $\mathfrak{FN}gMos$ in X.*

Proof. Let λ be a $\mathfrak{FN}os$ in Y. This implies λ^c is $\mathfrak{FN}cs$ in Y. Since f is $\mathfrak{FN}gMCts$, $f^{-1}(\lambda^c)$ is $\mathfrak{FN}gMcs$ in X. Since $f^{-1}(\lambda^c) = (f^{-1}(\lambda))^c$, $f^{-1}(\lambda)$ is a $\mathfrak{FN}gMos$ in X.

Conversely, let λ be a $\mathfrak{FN}os$ in Y. Then λ^c is a $\mathfrak{FN}cs$ in Y. By hypothesis $f^{-1}(\lambda^c)$ is $\mathfrak{FN}gMcs$ in X. Since $f^{-1}(\lambda^c) = (f^{-1}(\lambda))^c$, $(f^{-1}(\lambda))^c$ is a $\mathfrak{FN}gMcs$ in X. Therefore $f^{-1}(\lambda)$ is a $\mathfrak{FN}gMos$ in X. Hence f is $\mathfrak{FN}gMCts$.

Theorem 3.4. *Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ be a $\mathfrak{FN}gMCts$ map. Then the following conditions are hold.*

(i) $f(\mathfrak{FN}gMcl(\lambda)) \subseteq \mathfrak{FN}cl(f(\lambda))$, for all $\mathfrak{FN}cs$ λ in X.

(ii) $\mathfrak{FN}gMcl(f^{-1}(\mu)) \subseteq f^{-1}(\mathfrak{FN}cl(\mu))$, for all $\mathfrak{FN}cs$ μ in Y.

Proof. (i) Since $\mathfrak{FN}gMcl(f(\lambda))$ is a $\mathfrak{FN}gMcs$ in Y and f is $\mathfrak{FN}gMCts$, then $f^{-1}(\mathfrak{FN}gMcl(f(\lambda)))$ is $\mathfrak{FN}gMc$ in Y. Now, since $\lambda \subseteq f^{-1}(\mathfrak{FN}cl(f(\lambda)))$, $\mathfrak{FN}gMcl(\lambda)$ $\subseteq f^{-1}(\mathfrak{FN}gMcl(f(\lambda)))$. Therefore, $f(\mathfrak{FN}gMcl(\lambda)) \subseteq \mathfrak{FN}cl(f(\lambda))$.

(ii) By replacing λ with μ in (i), we obtain $f(\mathfrak{FN}gMcl(f^{-1}(\mu))) \subseteq \mathfrak{FN}cl(f(f^{-1}(\mu))) \subseteq \mathfrak{FN}cl(\mu)$. Hence, $\mathfrak{FN}gMcl\ (f^{-1}(\mu)) \subseteq f^{-1}(\mathfrak{FN}cl(\mu))$.

Remark 3.2. *If f is $\mathfrak{FN}gMCts$, then*

(i) $f(\mathfrak{FN}gMcl(\lambda))$ is not necessarily equal to $\mathfrak{FN}cl(f(\lambda))$ where $\lambda \in X$.

(ii) $\mathfrak{FN}gMcl(f^{-1}(\mu))$ is not necessarily equal to $f^{-1}(\mathfrak{FN}cl(\mu))$ where $\mu \in Y$.

Example 3.4. In Example 3.1, f is a $\mathfrak{FN}gMCts$.

(i) Let $\lambda = A_2$. Then $f(\mathfrak{F}\mathfrak{N}gMcl(\lambda)) = A_2$. But $\mathfrak{F}\mathfrak{N}cl(f(\lambda)) = A_3^c$. Thus $f(\mathfrak{F}\mathfrak{N}gMcl(\lambda)) \neq \mathfrak{F}\mathfrak{N}cl(f(\lambda))$.

(ii) Let $\lambda = A_1$. Then $f(\mathfrak{F}\mathfrak{N}gMcl(\lambda)) = A_3^c$. But $\mathfrak{F}\mathfrak{N}cl(f(\lambda)) = 1_{\mathfrak{F}\mathfrak{N}}$. Thus $f^{-1}(\mathfrak{F}\mathfrak{N}cl(\lambda)) \neq \mathfrak{F}\mathfrak{N}gMcl(f^{-1}(\lambda))$.

Theorem 3.5. *If f is $\mathfrak{F}\mathfrak{N}gMCts$, then $f^{-1}(\mathfrak{F}\mathfrak{N}int(\mu)) \subseteq \mathfrak{F}\mathfrak{N}gMint(f^{-1}(\mu))$, for all $\mathfrak{F}\mathfrak{N}cs$ μ in Y.*

Proof. If f is $\mathfrak{F}\mathfrak{N}gMCts$ and $\mu \in \sigma_{\mathfrak{F}\mathfrak{N}}$. $\mathfrak{F}\mathfrak{N}int(\mu)$ is $\mathfrak{F}\mathfrak{N}o$ in Y and hence, $f^{-1}(\mathfrak{F}\mathfrak{N}int(\mu))$ is $\mathfrak{F}\mathfrak{N}gMo$ in X. Therefore $\mathfrak{F}\mathfrak{N}gMint\ (f^{-1}(\mathfrak{F}\mathfrak{N}gMint(\mu))) = f^{-1}\ (\mathfrak{F}\mathfrak{N}int(\mu))$. Also, $\mathfrak{F}\mathfrak{N}int(\mu) \subseteq \mu$, implies that $f^{-1}(\mathfrak{F}\mathfrak{N}int(\mu)) \subseteq f^{-1}(\mu)$. Therefore $\mathfrak{F}\mathfrak{N}gMint\ (f^{-1}(\mathfrak{F}\mathfrak{N}int(\mu))) \subseteq \mathfrak{F}\mathfrak{N}gMint(f^{-1}(\mu))$. That is $f^{-1}(\mathfrak{F}\mathfrak{N}int(\mu)) \subseteq \mathfrak{F}\mathfrak{N}gMint\ (f^{-1}(\mu))$.

Conversely, let $f^{-1}(\mathfrak{F}\mathfrak{N}int(\mu)) \subseteq \mathfrak{F}\mathfrak{N}gMint(f^{-1}(\mu))$ for all subset μ of Y. If μ is $\mathfrak{F}\mathfrak{N}o$ in Y, then $\mathfrak{F}\mathfrak{N}int(\mu) = \mu$. By assumption, $f^{-1}(\mathfrak{F}\mathfrak{N}int(\mu)) \subseteq \mathfrak{F}\mathfrak{N}gMint(f^{-1}(\mu))$. Thus $f^{-1}(\mu) \subseteq \mathfrak{F}\mathfrak{N}gMint(f^{-1}(\mu))$. But $\mathfrak{F}\mathfrak{N}gMint(f^{-1}(\mu)) \subseteq f^{-1}(\mu)$. Therefore $\mathfrak{F}\mathfrak{N}gMint(f^{-1}(\mu)) = f^{-1}(\mu)$. That is, $f^{-1}(\mu)$ is $\mathfrak{F}\mathfrak{N}gMo$ in X, for all $\mathfrak{F}\mathfrak{N}os$ μ in Y. Therefore f is $\mathfrak{F}\mathfrak{N}gMCts$ on X.

Remark 3.3. *If f is $\mathfrak{F}\mathfrak{N}gMCts$, then $\mathfrak{F}\mathfrak{N}gMint(f^{-1}(\mu))$ is not necessarily equal to $f^{-1}(\mathfrak{F}\mathfrak{N}int(\mu))$ where $\mu \in Y$.*

Example 3.5. In Example 3.1, f is a $\mathfrak{F}\mathfrak{N}gMCts$. Let $\eta = A_1^c$. Then $f^{-1}(\mathfrak{F}\mathfrak{N}int(\eta)) = 0_{\mathfrak{F}\mathfrak{N}}$. But $\mathfrak{F}\mathfrak{N}gMint(f^{-1}(\eta)) = A_3$. Thus $f^{-1}(\mathfrak{F}\mathfrak{N}int(\eta)) \neq \mathfrak{F}\mathfrak{N}gMint(f^{-1}(\eta))$.

Definition 3.6. *For any two Fermatean neutrosophic subsets A and B, we shall write AqB to mean that A is Fermatean neutrosophic quasi-coincident with B if there exists $x \in X$ such that $\mu_A(x) + \mu_B(x) > 1$, $\nu_A(x) + \nu_B(x) > 1$ and $\sigma_A(x) + \sigma_B(x) < 1$.*

If A is not Fermatean neutrosophic quasi-coincident with B, then we write $A \not{q} B$.

Definition 3.7. *Let A and B be any two Fermatean neutrosophic subsets of a Fermatean neutrosophic topological space. Then A is a Fermatean neutrosophic q-neighbourhood with B if there exists a Fermatean neutrosophic open set O with $AqO \subseteq B$.*

Definition 3.8. *A Fermatean neutrosophic set A in a Fermatean neutrosophic topological space $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ is called a Fermatean neutrosophic generalized M q-neighbour -hood (briefly, $\mathfrak{F}\mathfrak{N}gMq$-nbhd) of a Fermatean neutrosophic point x_r if there exists a Fermatean neutrosophic generalized M open set V in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ such that $x_rqV \subseteq A$*

Proposition 3.1. *Let $h_{\mathfrak{F}} : (X_1, \tau_{\mathfrak{F}\mathfrak{N}}) \to (X_2, \sigma_{\mathfrak{F}\mathfrak{N}})$ be a Fermatean neutrosophic mapping between Fermatean neutrosophic topological spaces. Then the following assertions are equivalent.*

(i) $h_{\mathfrak{F}}$ is Fermatean neutrosophic generalized M continuous ($\mathfrak{F}\mathfrak{N}gMCts$).

(ii) For each Fermatean neutrosophic point $x_r \in X_1$ and every Fermatean neutrosophic generalized M q-neighbourhood A of $h_{\mathfrak{F}}(x_r)$, there exists a Fermatean neutrosophic generalized M open set B in X_1 such that $x_r \in B \subseteq h_{\mathfrak{F}}^{-1}(A)$.

(iii) For each Fermatean neutrosophic point $x_r \in X_1$ and every Fermatean neutrosophic generalized M q-neighbourhood A of $h_{\mathfrak{F}}(x_r)$, there exists a Fermatean neutrosophic generalized M open set B in X_1 such that $x_r \in B$ and $h_{\mathfrak{F}}(B) \subseteq A$.

Proof. (i) $\Rightarrow$ (ii) Let x_r be a Fermatean neutrosophic point in X_1 and let A be a Fermatean neutrosophic generalized M q-neighbourhood of $h_{\mathfrak{F}}(x_r)$. Then there exists a Fermatean neutrosophic generalized M open set B in X_2 such that $h_{\mathfrak{F}}(x_r) \in B \subseteq A$. Since $h_{\mathfrak{F}}$ is Fermatean neutrosophic generalized M continuous, we know that $h_{\mathfrak{F}}^{-1}(B)$ is a Fermatean neutrosophic generalized M open set in X_1 and $x_r \in h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}(x_r)) \subseteq h_{\mathfrak{F}}^{-1}(B) \subseteq h_{\mathfrak{F}}^{-1}(A)$. Consequently (ii) is valid.
(ii) $\Rightarrow$ (iii) Let x_r be a Fermatean neutrosophic point in X_1 and let A be a Fermatean neutrosophic generalized M q-neighbourhood of $h_{\mathfrak{F}}(x_r)$. The condition (ii) implies that there exists a Fermatean neutrosophic generalized M open set B in X_1 such that $x_r \in B \subseteq h_{\mathfrak{F}}^{-1}(A)$ so that $x_r \in B$ and $h_{\mathfrak{F}}(B) \subseteq h_{\mathfrak{F}}(h_{\mathfrak{F}}^{-1}(A)) \subseteq A$. Hence (iii) is true.
(iii) $\Rightarrow$ (i) Let B be a Fermatean neutrosophic open set in X_2 and let $x_r \in h_{\mathfrak{F}}^{-1}(B)$. Since B is a Fermatean neutrosophic open set, $h_{\mathfrak{F}}(x_r) \in B$, and so B is a Fermatean neutrosophic generalized M q-neighbourhood of $h_{\mathfrak{F}}(x_r)$. It follows from (iii) that there exists a Fermatean neutrosophic generalized M open set A in X_1 such that $x_r \in A$ and $h_{\mathfrak{F}}(A) \subseteq B$ so that $x_r \in A \subseteq h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}(A)) \subseteq h_{\mathfrak{F}}^{-1}(B)$. Applying Definition 3.8 induces that $h_{\mathfrak{F}}^{-1}(B)$ is a Fermatean neutrosophic generalized M open set in X_1. Therefore, $h_{\mathfrak{F}}$ is a Fermatean neutrosophic generalized M continuous function.

Remark 3.4. *The composition of two $\mathfrak{F}\mathfrak{N}gMCts$ functions need not be $\mathfrak{F}\mathfrak{N}gMCts$ as seen from the following example.*

Example 3.6. Let $X = Y = Z = \{a, b\}$ and the $\mathfrak{F}\mathfrak{N}s$'s A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4 and C_1 are defined as
$\mu_{A_1}(a) = 0.2,\ \nu_{A_1}(a) = 0.5,\ \sigma_{A_1}(a) = 0.8,$

$\mu_{A_1}(b) = 0.4,\ \nu_{A_1}(b) = 0.5,\ \sigma_{A_1}(b) = 0.6;$
$\mu_{A_2}(a) = 0.1,\ \nu_{A_2}(a) = 0.5,\ \sigma_{A_2}(a) = 0.9,$
$\mu_{A_2}(b) = 0.3,\ \nu_{A_2}(b) = 0.5,\ \sigma_{A_2}(b) = 0.7;$
$\mu_{A_3}(a) = 0.9,\ \nu_{A_3}(a) = 0.5,\ \sigma_{A_3}(a) = 0.1,$
$\mu_{A_3}(b) = 0.7,\ \nu_{A_3}(b) = 0.5,\ \sigma_{A_3}(b) = 0.3;$
$\mu_{A_4}(a) = 0.2,\ \nu_{A_4}(a) = 0.5,\ \sigma_{A_4}(a) = 0.8,$
$\mu_{A_4}(b) = 0.3,\ \nu_{A_4}(b) = 0.5,\ \sigma_{A_4}(b) = 0.7;$
$\mu_{B_1}(a) = 0.6,\ \nu_{B_1}(a) = 0.5,\ \sigma_{B_1}(a) = 0.4,$
$\mu_{B_1}(b) = 0.5,\ \nu_{B_1}(b) = 0.5,\ \sigma_{B_1}(b) = 0.5;$
$\mu_{B_2}(a) = 0.4,\ \nu_{B_2}(a) = 0.5,\ \sigma_{B_2}(a) = 0.6,$
$\mu_{B_2}(b) = 0.4,\ \nu_{B_2}(b) = 0.5,\ \sigma_{B_2}(b) = 0.6;$
$\mu_{B_3}(a) = 0.3,\ \nu_{B_3}(a) = 0.5,\ \sigma_{B_3}(a) = 0.7,$
$\mu_{B_3}(b) = 0.4,\ \nu_{B_3}(b) = 0.5,\ \sigma_{B_3}(b) = 0.6;$
$\mu_{B_4}(a) = 0.6,\ \nu_{B_4}(a) = 0.5,\ \sigma_{B_4}(a) = 0.4,$
$\mu_{B_4}(b) = 0.6,\ \nu_{B_4}(b) = 0.5,\ \sigma_{B_4}(b) = 0.4;$
$\mu_{C_1}(a) = 0.8,\ \nu_{C_1}(a) = 0.5,\ \sigma_{C_1}(a) = 0.2,$
$\mu_{C_1}(b) = 0.7,\ \nu_{C_1}(b) = 0.5,\ \sigma_{C_1}(b) = 0.3.$
Then we have $\tau_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, A_1, A_2, A_3, A_4, 1_{\mathfrak{FN}}\}$, $\sigma_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, B_1, B_2, B_3, B_4, 1_{\mathfrak{FN}}\}$ and $\eta_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, C_1, 1_{\mathfrak{FN}}\}$. Then, the identity mappings $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ and $g : (Y, \sigma_{\mathfrak{FN}}) \to (Z, \eta_{\mathfrak{FN}})$ are $\mathfrak{FN}gMCts$ but the composition $g \circ f$ is not $\mathfrak{FN}gMCts$, the set C_1^c is $\mathfrak{FN}c$ in Z but $(g \circ f)^{-1}(C_1^c) = C_1^c$ is not $\mathfrak{FN}gMcs$ in X.

Theorem 3.6. *Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ and $g : (Y, \sigma_{\mathfrak{FN}}) \to (Z, \nu_{\mathfrak{FN}})$ be any two functions. If f is a $\mathfrak{FN}gMCts$ and g is $\mathfrak{FN}$ Cts functions, then $g \circ f$ is $\mathfrak{FN}gMCts$.*
Proof. Let λ be any $\mathfrak{FN}c$ set in Z. As g is $\mathfrak{FN}$ Cts, $g^{-1}(\lambda)$ is $\mathfrak{FN}c$ in Y. Since f is $\mathfrak{FN}gMCts$, implies $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$ is $\mathfrak{FN}gMc$ in X. Therefore $g \circ f$ is $\mathfrak{FN}gMCts$.

Definition 3.9. *A function $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ is Fermatean neutrosophic generalized M^* continuous (briefly, $\mathfrak{FN}gM^*$ Cts, if for each $\mathfrak{FN}Mc$ set λ of Y, the set $f^{-1}(\lambda)$ is $\mathfrak{FN}gMc$ set in X.*

Theorem 3.7. *A function $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ is $\mathfrak{FN}gM^*$ Cts, iff $f^{-1}(\lambda)$ is $\mathfrak{FN}gMo$ in X for every $\mathfrak{FN}Mo$ set λ in Y*
Proof. The proof is similar in the Theorem 3.3.

Theorem 3.8. *Every $\mathfrak{FN}gM^*$ Cts function is $\mathfrak{FN}gMCts$. But converse need not be true in general.*
Proof. Proof follows from the fact that every $\mathfrak{FN}Mc$ set is $\mathfrak{FN}gMc$.

Example 3.7. Let $X = Y = \{a, b\}$ and the $\mathfrak{FN}s$'s A_1, A_2, A_3, A_4, B_1, B_2, B_3 and

B_4 are defined as
$\mu_{A_1}(a) = 0.2,\ \nu_{A_1}(a) = 0.5,\ \sigma_{A_1}(a) = 0.8,$
$\mu_{A_1}(b) = 0.4,\ \nu_{A_1}(b) = 0.5,\ \sigma_{A_1}(b) = 0.6;$
$\mu_{A_2}(a) = 0.1,\ \nu_{A_2}(a) = 0.5,\ \sigma_{A_2}(a) = 0.9,$
$\mu_{A_2}(b) = 0.3,\ \nu_{A_2}(b) = 0.5,\ \sigma_{A_2}(b) = 0.7;$
$\mu_{A_3}(a) = 0.9,\ \nu_{A_3}(a) = 0.5,\ \sigma_{A_3}(a) = 0.1,$
$\mu_{A_3}(b) = 0.7,\ \nu_{A_3}(b) = 0.5,\ \sigma_{A_3}(b) = 0.3;$
$\mu_{A_4}(a) = 0.2,\ \nu_{A_4}(a) = 0.5,\ \sigma_{A_4}(a) = 0.8,$
$\mu_{A_4}(b) = 0.3,\ \nu_{A_4}(b) = 0.5,\ \sigma_{A_4}(b) = 0.7;$
$\mu_{B_1}(a) = 0.6,\ \nu_{B_1}(a) = 0.5,\ \sigma_{B_1}(a) = 0.4,$
$\mu_{B_1}(b) = 0.5,\ \nu_{B_1}(b) = 0.5,\ \sigma_{B_1}(b) = 0.5;$
$\mu_{B_2}(a) = 0.4,\ \nu_{B_2}(a) = 0.5,\ \sigma_{B_2}(a) = 0.6,$
$\mu_{B_2}(b) = 0.4,\ \nu_{B_2}(b) = 0.5,\ \sigma_{B_2}(b) = 0.6;$
$\mu_{B_3}(a) = 0.3,\ \nu_{B_3}(a) = 0.5,\ \sigma_{B_3}(a) = 0.7,$
$\mu_{B_3}(b) = 0.4,\ \nu_{B_3}(b) = 0.5,\ \sigma_{B_3}(b) = 0.6;$
$\mu_{B_4}(a) = 0.6,\ \nu_{B_4}(a) = 0.5,\ \sigma_{B_4}(a) = 0.4,$
$\mu_{B_4}(b) = 0.6,\ \nu_{B_4}(b) = 0.5,\ \sigma_{B_4}(b) = 0.4.$

Then we have $\tau_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, B_1, B_2, B_3, B_4, 1_{\mathfrak{FN}}\}$ and $\sigma_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, A_1, A_2, A_3, A_4, 1_{\mathfrak{FN}}\}$. Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ be an identity mapping, then f is $\mathfrak{FN}gMCts$ but not $\mathfrak{FN}gM^*$, the set A_3 is a $\mathfrak{FN}Mcs$ in Y but $f^{-1}(A_3)$ is not $\mathfrak{FN}gMcs$ in X.

Remark 3.5. *The composition of two $\mathfrak{FN}gM^*$ Cts functions need not be $\mathfrak{FN}gM^*$ Cts as seen from the following example.*

Example 3.8. In Example 3.6, the identity mappings $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ and $g : (Y, \sigma_{\mathfrak{FN}}) \to (Z, \eta_{\mathfrak{FN}})$ are $\mathfrak{FN}gM^*$ Cts but the composition $g \circ f$ is not $\mathfrak{FN}gM^*$ Cts, the set C_1^c is $\mathfrak{FN}Mc$ in Z but $(g \circ f)^{-1}(C_1^c) = C_1^c$ is not $\mathfrak{FN}gMcs$ in X.

Theorem 3.9. *Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ and $g : (Y, \sigma_{\mathfrak{FN}}) \to (Z, \nu_{\mathfrak{FN}})$ be any two functions. If f is a $\mathfrak{FN}gM^*$ Cts and g is $\mathfrak{FN}MCts$ functions, then $g \circ f$ is $\mathfrak{FN}gMCts$.*

Proof. Let λ be any $\mathfrak{FN}Mc$ set in Z. As g is $\mathfrak{FN}MCts$, $g^{-1}(\lambda)$ is $\mathfrak{FN}Mc$ in Y. Since f is $\mathfrak{FN}gM^*$ Cts, implies $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$ is $\mathfrak{FN}gMc$ in X. Therefore $g \circ f$ is $\mathfrak{FN}gMCts$.

4. Fermatean neutrosophic generalized M-irresolute maps

In this section we introduce Fermatean neutrosophic generalized M-irresolute maps and study some of its characterizations.

Definition 4.1. *A map $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ is called a Fermatean neutrosophic generalized*

(i) *irresolute (briefly,* $\mathfrak{FN}gIrr$*) map if* $f^{-1}(\lambda)$ *is a* $\mathfrak{FN}gcs$ *in* $(X, \tau_{\mathfrak{FN}})$ *for every* $\mathfrak{FN}gcs$ λ *of* $(Y, \sigma_{\mathfrak{FN}})$,

(ii) θ*-irresolute (briefly,* $\mathfrak{FN}g\theta Irr$*) map if* $f^{-1}(\lambda)$ *is a* $\mathfrak{FN}g\theta cs$ *in* $(X, \tau_{\mathfrak{FN}})$ *for every* $\mathfrak{FN}g\theta cs$ λ *of* $(Y, \sigma_{\mathfrak{FN}})$,

(iii) $\theta\mathcal{S}$*-irresolute (briefly,* $\mathfrak{FN}g\theta\mathcal{S}Irr$*) map if* $f^{-1}(\lambda)$ *is a* $\mathfrak{FN}g\theta\mathcal{S}cs$ *in* $(X, \tau_{\mathfrak{FN}})$ *for every* $\mathfrak{FN}g\theta\mathcal{S}cs$ λ *of* $(Y, \sigma_{\mathfrak{FN}})$,

(iv) $\delta\mathcal{P}$*-irresolute (briefly,* $\mathfrak{FN}g\delta\mathcal{P}Irr$*) map if* $f^{-1}(\lambda)$ *is a* $\mathfrak{FN}g\delta\mathcal{P}cs$ *in* $(X, \tau_{\mathfrak{FN}})$ *for every* $\mathfrak{FN}g\delta\mathcal{P}cs$ λ *of* $(Y, \sigma_{\mathfrak{FN}})$,

(v) M*-irresolute (briefly,* $\mathfrak{FN}gMIrr$*) map if* $f^{-1}(\lambda)$ *is a* $\mathfrak{FN}gMcs$ *in* $(X, \tau_{\mathfrak{FN}})$ *for every* $\mathfrak{FN}gMcs$ λ *of* $(Y, \sigma_{\mathfrak{FN}})$.

Theorem 4.1. *A function* $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ *is* $\mathfrak{FN}gMIrr$ *(resp.* $\mathfrak{FN}gIrr$, $\mathfrak{FN}g\theta Irr$, $\mathfrak{FN}g\theta\mathcal{S}Irr$ *and* $\mathfrak{FN}g\delta\mathcal{P}Irr$*) mapping, then* f *is* $\mathfrak{FN}gMCts$ *(resp.* $\mathfrak{FN}gCts$, $\mathfrak{FN}g\theta Cts$, $\mathfrak{FN}g\theta\mathcal{S}Cts$ *and* $\mathfrak{FN}g\delta\mathcal{P}Cts$*). But the converse does not true.*
Proof. Let λ be a $\mathfrak{FN}cs$ in Y, then λ is $\mathfrak{FN}gMcs$ in Y. Since every $\mathfrak{FN}cs$ is $\mathfrak{FN}gMcs$. Since f is $\mathfrak{FN}gMIrr$, $f^{-1}(\lambda)$ is a $\mathfrak{FN}gMcs$ in X. Hence f is a $\mathfrak{FN}gMCts$.

Example 4.1. In Example 3.7, let $f : (X, \sigma_{\mathfrak{FN}}) \to (Y, \tau_{\mathfrak{FN}})$ be an identity mapping, then$f$ is $\mathfrak{FN}gMCts$ but not $\mathfrak{FN}gMIrr$, the set A_4 is a $\mathfrak{FN}gMcs$ in Y but $f^{-1}(A_4)$ is not $\mathfrak{FN}gMcs$ in X.

Theorem 4.2. *A map* $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ *is* $\mathfrak{FN}gMIrr$ *iff the inverse image of each* $\mathfrak{FN}Mos$ *in* Y *is* $\mathfrak{FN}gMos$ *in* X.
Proof. Let λ be a $\mathfrak{FN}Mos$ in Y. This implies λ^c is $\mathfrak{FN}Mcs$ in Y. Since f is $\mathfrak{FN}gMIrr$, $f^{-1}(\lambda^c)$ is $\mathfrak{FN}gMcs$ in X. Since $f^{-1}(\lambda^c) = (f^{-1}(\lambda))^c$, $f^{-1}(\lambda)$ is a $\mathfrak{FN}gMos$ in X.

Conversely, let λ be a $\mathfrak{FN}Mos$ in Y. Then λ^c is a $\mathfrak{FN}Mcs$ in Y. By hypothesis $f^{-1}(\lambda^c)$ is $\mathfrak{FN}gMcs$ in X. Since $f^{-1}(\lambda^c) = (f^{-1}(\lambda))^c$, $(f^{-1}(\lambda))^c$ is a $\mathfrak{FN}gMcs$ in X. Therefore $f^{-1}(\lambda)$ is a $\mathfrak{FN}gMos$ in X. Hence f is $\mathfrak{FN}gMIrr$.

Theorem 4.3. *Let* $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ *and* $g : (Y, \sigma_{\mathfrak{FN}}) \to (Z, \nu_{\mathfrak{FN}})$ *be any two functions. If* f *and* g *is a* $\mathfrak{FN}gMIrr$ *functions, then* $g \circ f$ *is* $\mathfrak{FN}gMIrr$.
Proof. Let λ be any $\mathfrak{FN}gMc$ set in Z. As g is $\mathfrak{FN}gMIrr$, $g^{-1}(\lambda)$ is $\mathfrak{FN}gMc$ in Y. Since f is $\mathfrak{FN}gMIrr$, implies $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$ is $\mathfrak{FN}gMc$ in X. Therefore $g \circ f$ is $\mathfrak{FN}gMIrr$.

Theorem 4.4. *Let* $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ *and* $g : (Y, \sigma_{\mathfrak{FN}}) \to (Z, \nu_{\mathfrak{FN}})$ *be any two functions.*

1. *If f is a $\mathfrak{FN}gMIrr$ and g is $\mathfrak{FN}gMCts$ functions, then $g \circ f$ is $\mathfrak{FN}gMCts$.*

2. *If f is a $\mathfrak{FN}gMCts$ and g is $\mathfrak{FN}$ Cts functions, then $g \circ f$ is $\mathfrak{FN}gMCts$.*

Proof. (i) Let λ be any $\mathfrak{FN}c$ set in Z. As g is $\mathfrak{FN}gMCts$, $g^{-1}(\lambda)$ is $\mathfrak{FN}gMc$ in Y. Since f is $\mathfrak{FN}gMIrr$, implies $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$ is $\mathfrak{FN}gMc$ in X. Therefore $g \circ f$ is $\mathfrak{FN}gMCts$. (ii) Let λ be any $\mathfrak{FN}c$ set in Z. As g is $\mathfrak{FN}$ Cts, $g^{-1}(\lambda)$ is $\mathfrak{FN}c$ in Y. Since f is $\mathfrak{FN}gMCts$, implies $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$ is $\mathfrak{FN}gMc$ in X. Therefore $g \circ f$ is $\mathfrak{FN}gMCts$.

Theorem 4.5. *Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ and $g : (Y, \sigma_{\mathfrak{FN}}) \to (Z, \nu_{\mathfrak{FN}})$ be any two functions. If f is a $\mathfrak{FN}gMIrr$ and g is $\mathfrak{FN}MCts$ functions, then $g \circ f$ is $\mathfrak{FN}gMCts$.*

Proof. Let λ be any $\mathfrak{FN}c$ set in Z. As g is $\mathfrak{FN}MCts$, $g^{-1}(\lambda)$ is $\mathfrak{FN}Mc$ in Y. Since f is $\mathfrak{FN}gMIrr$, implies $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$ is $\mathfrak{FN}gMc$ in X. Therefore $g \circ f$ is $\mathfrak{FN}gMCts$.

Theorem 4.6. *Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ and $g : (Y, \sigma_{\mathfrak{FN}}) \to (Z, \nu_{\mathfrak{FN}})$ be any two functions. If f is a $\mathfrak{FN}gMIrr$ and g is $\mathfrak{FN}g$ Cts functions, then $g \circ f$ is $\mathfrak{FN}gMCts$.*

Proof. Let λ be any $\mathfrak{FN}c$ set in Z. As g is $\mathfrak{FN}$ Cts, $g^{-1}(\lambda)$ is $\mathfrak{FN}c$ in Y. Since f is $\mathfrak{FN}gMIrr$, implies $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$ is $\mathfrak{FN}gMc$ in X. Therefore $g \circ f$ is $\mathfrak{FN}gMCts$.

Theorem 4.7. *Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ be a mapping. If f is*

(i) $\mathfrak{FN}gMIrr$, then it is $\mathfrak{FN}gM^*Cts$,

(ii) $\mathfrak{FN}gM^*Cts$, then it is $\mathfrak{FN}MCts$.

Proof. The proof is immediate.

Remark 4.1. *Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ be a mapping. If f is $\mathfrak{FN}MIrr$, then it is $\mathfrak{FN}gM^*Cts$. But converse need not be true in general.*

Example 4.2. In Example 3.7, f is $\mathfrak{FN}gM^*Cts$ but not $\mathfrak{FN}MIrr$, the set A_3 is a $\mathfrak{FN}Mcs$ in Y but $f^{-1}(A_3)$ is not $\mathfrak{FN}Mcs$ in X.

Theorem 4.8. *Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ and $g : (Y, \sigma_{\mathfrak{FN}}) \to (Z, \nu_{\mathfrak{FN}})$ be any two functions. If f is a $\mathfrak{FN}gM^*$ Cts and g is $\mathfrak{FN}MIrr$ functions, then $g \circ f$ is $\mathfrak{FN}gM^*$ Cts.*

Proof. Let λ be any $\mathfrak{FN}Mc$ set in Z. As g is $\mathfrak{FN}MIrr$, $g^{-1}(\lambda)$ is $\mathfrak{FN}Mc$ in Y. Since f is $\mathfrak{FN}gM^*$ Cts, implies $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$ is $\mathfrak{FN}gMc$ in X. Therefore $g \circ f$ is $\mathfrak{FN}gM^*$ Cts.

Theorem 4.9. *Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ and $g : (Y, \sigma_{\mathfrak{FN}}) \to (Z, \nu_{\mathfrak{FN}})$ be any two functions. If f is a $\mathfrak{FN}gMIrr$ and g is $\mathfrak{FN}gM^*$ Cts functions, then $g \circ f$ is $\mathfrak{FN}gM^*$ Cts.*

Proof. Let λ be any $\mathfrak{FN}Mc$ set in Z. As g is $\mathfrak{FN}gM^*$ Cts, $g^{-1}(\lambda)$ is $\mathfrak{FN}gMc$ in Y. Since f is $\mathfrak{FN}gMIrr$, implies $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$ is $\mathfrak{FN}gMc$ in X. Therefore $g \circ f$ is $\mathfrak{FN}gM^*$ Cts.

5. Conclusion

The primary goal of this research is to extend the theory of Fermatean Neutrosophic topology by introducing and systematically analyzing two novel classes of generalized mappings: Fermatean Neutrosophic generalized M-continuous ($\mathfrak{FN}gM$-Cts) and Fermatean Neutrosophic generalized M-irresolute ($\mathfrak{FN}gMIrr$) mappings. While grounded in classical topological ideas, our work demonstrates that these new concepts provide a richer and more flexible framework for handling uncertainty and vagueness in Fermatean Neutrosophic settings.

The principal results of this study are fourfold:

(i) Fundamental Properties of $\mathfrak{FN}gM$-continuous mappings: We established that $\mathfrak{FN}gM$-continuity is preserved under composition, a cornerstone property that validates the robustness of the proposed framework. These mappings satisfy desirable topological axioms and exhibit behavior consistent with classical continuous functions, while simultaneously generalizing several existing notions of Fermatean neutrosophic continuity.

(ii) Characterization of $\mathfrak{FN}gM$-irresolute mappings: The introduction of $\mathfrak{FN}g$ $MIrr$ mappings and their characterization in terms of $\mathfrak{FN}gM$-closed sets reveals new structural properties. Notably, these mappings preserve generalized M-closed sets, ensuring that topological structures are maintained under mappings, a critical feature for applications.

(iii) Composition Properties and Interrelations: We proved several composition theorems demonstrating that compositions of $\mathfrak{FN}gM$-continuous and $\mathfrak{FN}gMIrr$ mappings retain their continuity properties. These results establish a hierarchical relationship among different classes of mappings, enhancing the overall theoretical coherence of the framework.

(iv) Illustrative examples and structural insights: Through carefully constructed examples and detailed proofs, we demonstrated that the proposed mappings are genuinely distinct from existing generalized continuity concepts in Fermatean neutrosophic topology. The examples validate our theoretical results and provide concrete instantiations of the abstract concepts.

Significance and Theoretical Impact: The significance of this work extends beyond incremental theoretical contributions. First, our framework unifies and extends multiple existing notions of continuity and irresoluteness in fuzzy and neutrosophic topologies, providing a more general and powerful tool for researchers. Second, the generalized M-open sets upon which these mappings are based capture both membership and non-membership information in a Fermatean manner (where the third power of degrees sums to at most one), offering superior representational capacity compared to intuitionistic or Pythagorean fuzzy approaches.

Third, the preservation of topological structures under the proposed mappings has direct implications for topology-based applications. In particular, the results establish that Fermatean neutrosophic topology is stable under natural operations, a prerequisite for reliability in applied settings. The employment of generalized M-sets enables more nuanced treatment of boundary phenomena, indeterminacy, and degree-theoretic aspects critical to real-world decision-making scenarios.

Fourth, this investigation fills a notable gap in the existing literature by providing the first comprehensive treatment of generalized M-mappings in the Fermatean Neutrosophic context, thereby advancing the state of knowledge in fuzzy and neutrosophic topology.

Implications for Applications: Beyond their theoretical importance, the results of this study provide a solid mathematical foundation for applications in diverse fields including decision sciences, medical diagnosis, pattern recognition, and machine learning, where uncertainty and vagueness are inherent. The greater flexibility offered by Fermatean Neutrosophic frameworks compared to classical and intuitionistic fuzzy approaches makes the proposed mappings particularly valuable for complex, multi-attribute decision problems and information systems requiring fine-grained uncertainty quantification.

Future Directions: Future research could profitably explore: (1) the introduction of separation axioms and compactness properties in the context of $\mathfrak{F}\mathfrak{N}gM$-continuous mappings; (2) the application of these mappings to topology - based MAGDM (Multi-Attribute Group Decision Making) problems; (3) the investigation of connectedness and related properties with respect to these generalized mappings; and (4) extensions to hybrid frameworks combining Fermatean Neutrosophic topology with other uncertainty models such as interval-valued or hesitant fuzzy structures.

In conclusion, this work provides a strong theoretical foundation for further developments in Fermatean Neutrosophic topology and opens promising avenues for both theoretical research and practical applications in modern data analysis and decision-making systems.

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